Periodic Scheduling on Heterogeneous Resources

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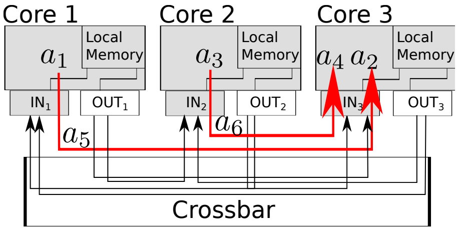
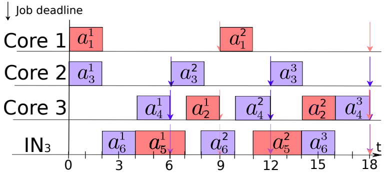
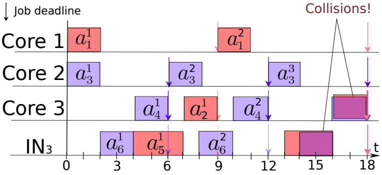
Motivation example - inspired by Infineon AURIX TriCore



- assignment of tasks is given
- period of $a_1 \to a_5 \to a_2$ is 9
- period of $a_3 \to a_6 \to a_4$ is 6
- release dates - starts of period
- deadlines - ends of period



Schedule with jitter-constrains.



Schedule with zero-jitter.

Problem Statement - Periodic Extension of $PSm, 1\,|\text{prec}|-$

- The goal is to find **a feasible schedule with a hyper-period** $H = lcm(\tau_1, \tau_2, \cdots, \tau_n)$.
- An assignment of tasks is given.
- The schedule is defined by:
  start times $s_i^j \in \mathbb{N}$ of j-th **occurence of task** $T_i \in \mathbb{T}$ where $j = 1, 2, \cdots, n_i$, and $n_i = \frac{H}{\tau_i}$.
- The schedule must satisfy:
  - periodic nature of the tasks
  - precedence relations given by DAG
  - jitter constraints
  - release dates
  - deadlines

The considered scheduling problem can be categorized as *multi-periodic non-preemptive scheduling of tasks with precedence and jitter constraints on dedicated resources of capacity one*. We deal with a decision problem.

Jitter-Constrained Schedule and Zero-Jitter Schedule

Definition (Zero-Jitter (ZJ) schedule)

The schedule is a ZJ schedule if and only if for each task T_i the following equation is valid,

$$s_i^{j+1} - (s_i^j + \tau_i) = 0 \quad \forall j = 1, 2, \cdots, n_i - 1$$

i.e. the difference between the start times s_i^j and s_i^{j+1} in each pair of consecutive occurencies j and $j+1$ over the hyper-period is the same and equal to the period.

Definition (Jitter-Constrained (JC) Schedule)

The schedule is a JC schedule if and only if for each task T_i the following equations are valid,

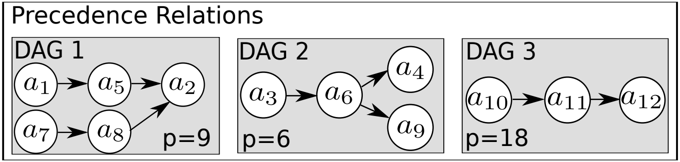
$$|s_i^{j+1} - (s_i^j + \tau_i)| \leq jit_i \quad \forall j = 1, 2, \cdots, n_i - 1$$

$$|s_i^1 + H - (s_i^{n_i} + \tau_i)| \leq jit_i$$

Constraints

- **precedence constraints** based on DAG give by the set of edges E

$$s_i^j + p_i \leq s_l^j \qquad \forall T_i,\ T_l \in \mathbb{T} : \exists edge\ e_{i,l} \in E,\ \forall j = 1, \cdots, n_i$$



- **release date and deadline constraints** and it requires each activity to be executed in a given time interval of **two periods**

$$(j-1) \cdot \tau_i \leq s_i^j \leq (j+1) \cdot \tau_i - p_i \qquad \forall T_i \in \mathbb{T},\ \forall j = 1, \cdots, n_i$$

Resource Constraints

We introduce a binary decision variable,

$$x_{i,l}^{j,k} = \begin{cases} 1, & \text{if } T_i^j \text{ starts before } T_l^k \\ 0, & \text{otherwise.} \end{cases}$$

to avoid collision of two taks on the same resource.

$$\begin{aligned} s_i^j + p_i \leq & \quad s_l^k + 2 \cdot H \cdot (1 - x_{i,l}^{j,k}) \\ s_l^k + p_l \leq & \quad s_i^j + 2 \cdot H \cdot x_{i,l}^{j,k} \\ \forall T_i,\ T_l \in \mathbb{T} \text{ assigned to} & \quad \text{the same resource}, \forall j = 1, ..., n_i,\ k = 1, ..., n_l \end{aligned}$$

Every task occurrence has the time window of $2 \cdot \tau_i$. Consequently:

- in above equation the Big M constant is equal to $2 \cdot H$;
- we have to avoid collision of two subsequent occurrences of the task:

$$s_i^j + p_i \leq s_i^{j+1}$$

$$s_i^{n_i} + p_i \leq s_i^0 + H$$

$$\forall T_i \in \mathbb{T},\ \forall j = 1, \cdots, n_i - 1$$

Jitter Constrains

- We consider a **relative jitter**, where we bound only the difference in start times of task occurrences in consecutive periods.
- To formulate the jitter constraints in a linear form, the inequality with the **absolute value** $|z| \leq konst$ needs to be **replaced** by $z \leq konst$ and $z \geq -konst$.
- We obtain two constraints for the **jobs inside** one hyper-period and two for the **jobs on the border**.

$$
\begin{aligned}
s_i^{j+1} - (s_i^j + \tau_i) \leq jit_i, \\
s_i^{j+1} - (s_i^j + \tau_i) \geq -jit_i \\
s_i^1 + H - (s_i^{n_i} + \tau_i) \leq jit_i \\
s_i^1 + H - (s_i^{n_i} + \tau_i) \geq -jit_i \\
\forall T_i \in \mathbb{T},\ \forall j = 1, \cdots, n_i - 1
\end{aligned}
$$

Complete Study and Evaluation of ZJ vs. JC

Minaeva, A - Akesson, B. - Hanzálek, Z. - Dasari, D.: Time-Triggered Co-Scheduling of Computation and Communication with Jitter Requirements, IEEE Transactions on Computers, Volume 67, Issue 1, Jan. 2018 , Pages 115-129, doi: 10.1109/TC.2017.2722443.

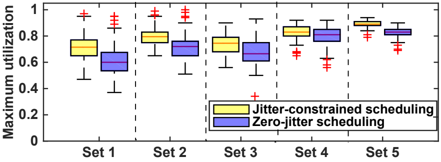


Figure: Maximum utilization distribution for the 3-LS heuristic with JC and ZJ requirements in sets with 20, 30, 50, 100 and 500 tasks. The average difference between JC and ZJ is 15.3%, 9.7%, 8.6%, 4.2% and 7.5%

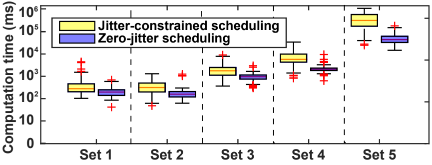


Figure: Computation time distribution for the 3-LS heuristic with JC and ZJ requirements in sets with 20, 30, 50, 100 and 500 tasks.