MODELING OF SYSTEMS WITH DELAYS USING HYBRID PETRI NETS

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Abstract: In practice the systems are not only pure discrete event or pure continuous, but they are combined from discrete and continuous parts together. These systems are called hybrid systems. Hybrid Petri nets (HPNs) allow modeling and analysis of hybrid systems. Since classical HPNs cannot properly model the systems with the delays on continuous product flow, extended Hybrid Petri nets and generalized batches Petri nets (GBPNs) were defined. This paper deals with extended HPNs and GBPNs, which are used in practical examples elaborated in this paper. *Copyright © 2003 IFAC*

Key words: Delay, Hybrid Petri-nets, Extended Hybrid Petri-nets, Batches Petri-nets, Modelling

1. INTRODUCTION

Petri nets (PNs) (Murata, 1989) are used to model and analyze discrete event dynamic systems like manufacturing systems and communication protocols. In continuous Petri nets (CPNs), the markings of places are real numbers and the firing of transitions is a continuous process (David and Alla, 1993). Therefore CPNs allow modeling and analysis of continuous systems. Moreover CPNs can be used to approximate behavior of discrete systems represented by discrete PN with large number of tokens. In such case, analysis of CPNs does not require exhaustive enumeration of the discrete state space.

In practice the systems are not only pure discrete event or pure continuous, but they are combined from discrete and continuous parts together (discrete state variables and continuous state variables). Hybrid Petri nets (HPNs) are one of the tools, which allow modeling of such systems (David and Alla, 1992; David and Alla, 2001). A timed model of HPNs may be used for performance evaluation of the systems. Timed HPNs have inherited various modeling abilities from the previous models related to time. In the classical model adapted by David and Alla (1998), the discrete part of HPN enables to model delay with threshold and the continuous part of HPN models the continuous flows without threshold and without delay.

Beside of that there exist the systems with delays on the continuous product flow (e.g. the product delay on the conveyor, or delay of fluid in pipe). Since there is no threshold of the product volume, the delay on continuous product flow cannot be properly modeled by HPNs. Several authors have proposed extensions of the original HPNs model in order to represent delays on the continuous flows. David and Caramihai (2000) have shown use of extended HPNs, Demongodin (2001) defined generalized batches Petri nets and Brinkman and Blaauboer (2000) assigned delays to continuous places. Extended HPNs and generalized batches Petri nets are used in practical examples elaborated in this text.

This paper is organized as follows: Section 2 briefly presents HPNs, followed by introduction to extended HPNs and generalized batches Petri nets (GBPNs). Section 3 shows various possibilities of modeling systems with the delay on continuous flow. Section 4 presents example with delay on continuous flow: delay in physical layer of communication protocol.

2. DESCRIPTION OF HYBRID PETRI NETS

In this section hybrid Petri nets are defined first. Next basic concepts of extended hybrid Petri nets and generalized batches Petri nets are presented. The instantaneous firing speeds in the continuous part of HPNs are supposed to be constrained by constant values. This model corresponds to classical constant speed continuous Petri Net (CCPN).

A marked timed HPN net is defined by the seven-tuple:

$$\langle \mathbf{P}, \mathbf{T}, Pre, Post, M_0, V, d \rangle \quad (1)$$

$\mathbf{P} = P \cup CP$	finite and non-empty set of places (P and CP are disjoint)
$\mathbf{T} = T \cup CT$	finite and non-empty set of transitions (T and CT are disjoint)
Pre: $\mathbf{P}\mathrm{x}\mathbf{T} \rightarrow R^+$ or N	input incidence matrix
Post: $\mathbf{P}\mathrm{x}\mathbf{T} \rightarrow R^+$ or N	output incidence matrix
M_0: $\mathbf{P} \rightarrow R^+$ or N	vector of initial marking
$V=\{V_1,..,V_m\}$	vector of maximal speeds
$d=\{d_1,..,d_n\}$	vector of timings

In a definition of Pre, Post and M_0, R^+ corresponds to case, when $\mathbf{P} \in CP$ and N represents case, when $\mathbf{P} \in P$.

HPN contains the set of discrete places and transitions (P_i, T_j - where P_i is the discrete place and T_j is the discrete transition) and the set of continuous places and transitions (CP_i, CT_j - where CP_i is continuous place and CT_j is continuous transition). The discrete part and the continuous part influence each other depending on firing rules of discrete and continuous transitions (David and Alla, 1998). The continuous place CP_i can be input or output place of a discrete transition T_j without restriction. T_j is not enabled if the marking of its input place CP_i is lower than $Pre(CP_i, T_j)$. This value represents the threshold of continuous marking. Timed HPN has the delay d_j associated to discrete transitions T_j and the maximal firing speed V_j associated to continuous transition CT_j (calculation of instantaneous firing speed is presented in (David and Alla, 1993). The delay d_j expresses how long the needed quantity (in the case of continuous input place) or the number of tokens (in the case of discrete input places) is reserved. Being enabled during d_j time units the discrete transition T_j is fired.

HPNs can be studied using standard PN analysis techniques such as invariants. Incidence matrix W of HPN is combined from discrete part W^D, continuous part W^C and the part representing influence of continuous places on discrete transitions W^{CD}. The incidence matrix W does not reflect influence of any discrete place P_i on any continuous transition CT_j since $Pre(P_i, CT_j) = Post(P_i, CT_j)$.

$$W = Post - Pre = \begin{pmatrix} W^D & 0 \\ W^{CD} & W^C \end{pmatrix} \quad (2)$$

In HPNs, characteristic vector ***s*** of a sequence is a vector for which each element is either non-negative real number corresponding to the firing quantity of a continuous transition CT_j or a positive integer number corresponding to the number of firings of a discrete transition T_j. A marking M can be calculated from initial marking M_0 and its appropriate characteristic vector ***s*** using the fundamental equation (3):

$$M = M_0 + W \cdot s \quad (3)$$

2.1 Evolution graph of Hybrid Petri nets

An evolution graph of HPN is an oriented graph consisting of nodes corresponding to IB-states (invariant behavior states) and transitions between these nodes. The IB-state characterizes the state of HPN during certain time. The marking of discrete places and the instantaneous firing speed of continuous transitions remain constant as long as the system is in the same IB-state. The transition of the evolution graph corresponds to event provoking passage from one IB-state to another IB-state. It is labeled by the name of the event, the time elapsed in previous IB-state and the absolute time.

An IB-state consists of two parts in HPNs:

- the discrete part on the left side of the IB-state is given by M_D (the marking of discrete places).
- the continuous part on the right side of the IB-state is given by v_C, the instantaneous speed of continuous transitions and by M_C, the marking of continuous places at the beginning of the IB-state.

Timed HPNs have inherited various modeling abilities from discrete PNs and CPNs, which are related to time. Therefore the discrete part of HPN enables to model delay with threshold and the continuous part of HPN models the continuous flows without threshold and without delay. Beside of that there exist the systems with delays on the continuous product flow (e.g. the product delay on the conveyor, or delay of fluid in pipe). Since there is no threshold of the product volume, the continuous product flow delay cannot be properly modeled by HPNs.

2.2 Extended hybrid Petri nets

The extended HPNs (David and Caramihai, 2000) have the following features in addition to ordinary HPNs:

- an inhibitor arc is allowed
- 0^+ weight of an arc joining a continuous place and discrete transition is allowed
- 0^+ marking of continuous place is allowed

Symbol 0^+ represents infinitely small positive real number. Please notice that due to the second feature we are able to model delays on continuous flows by extended HPNs.

2.3 Generalized Batches Petri nets

Generalized batches Petri nets (GBPNs) (Demongodin, 2001) are another extension of HPNs.

New elements are batch places and batch transitions. Batch place allows to model delays on continuous flows. It is described by the batch function, which is given by speed of transfer, maximal density and length of place $\{V_i, d_{max}, s_i\}$. For each batch place, the set of input transitions consists of only one batch transition and the set of output transitions consists of only one batch transition as well. Therefore there is no structural conflict related to any batch place. The batch transition behaves like the continuous transition.

Important notion in GBPN is internal coherent batch ($ICBp_i$) assigned to the beginning and to the end of the IB-state. The marking of batch place BP_i is given by an ordered sequence of $ICBp_i$. Index p of $ICBp_i$ expresses its placing in the ordered sequence of the batch place BP_i. At fixed time t, ICBp is characterized by three continuous variables: length $l_p(t)$, density $d_p(t)$ and position $x_p(t)$ in batch place. If position of the batch is equal to the characterized length of batch place s_i, the batch becomes the output internal coherent batch (OICBp).

3. EXAMPLES

Let us now present a simple example modeled in three distinct ways: by discrete PN, by extended HPN and by GBPN. Fig. 1 represents conveyor with an input buffer B_1 and an output buffer B_2. In initial state, the input buffer B_1 contains 10 kilograms of a product and the output buffer B_2 is empty. The product is transferred via conveyor from B1 to B_2. The conveyor length L is 9 meters and its speed V is 0.5 m/sec. Maximal density D_{max} of the product on the conveyor is 1 kg/m.

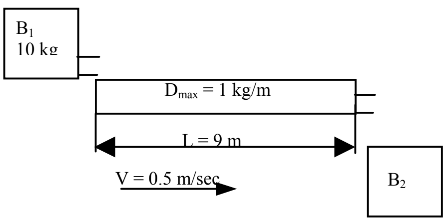

Fig. 1. – Model of conveyor

3.1 Discrete PN model

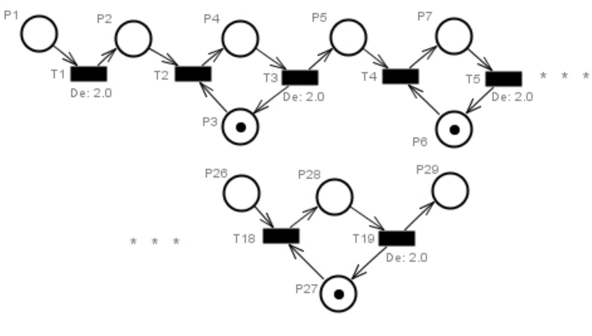

Fig. 2. – Conveyor model by discrete PN

Conveyor model by the discrete PN is shown in Fig. 2. The mode is based on the single server semantics. Input buffer is represented by place P1. Each kilogram of the product is supposed to be a discrete quantity therefore P1 contains 10 tokens. Timing of transition T1, having the threshold Pre(P1,T1)=1, represents a delay needed to put 1 kilogram of the product on the conveyor. T1 fires for the first time when the first kilogram leaves the input buffer B1. Timed transitions T3, T5, … T19 correspond to the delay on each meter of the conveyor. Places P3, P6, P27 represent that the given part (1 meter) of the conveyor is empty. Places P4, P7,…P28 represent that the given part of the conveyor is full. Between each couple of timed transitions there is one immediate transition (one of T2,T4,… T18), otherwise the timed behavior of the model does not correspond to reality, since the "empty part" of the conveyor (places P3, P6,…P27) is logical and not timed constraint when moving the product from one part to the subsequent one.

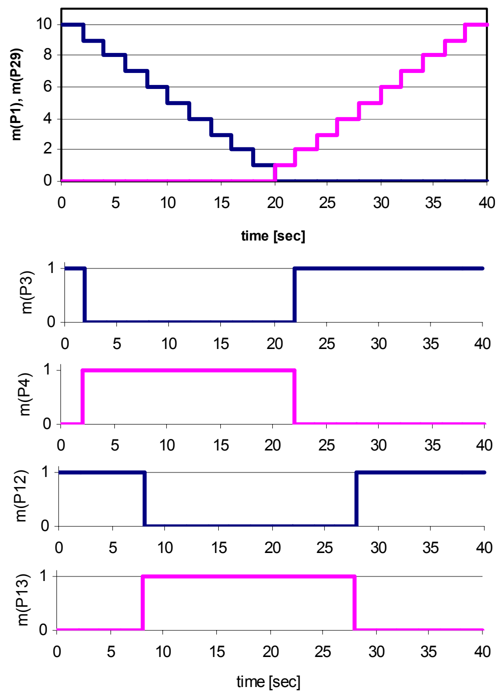

Fig. 3. – The marking evolution of model in Fig. 2

Since the state space of the modeled system is too large, the marking evolution of several places is illustrated in Fig. 3. The first figure represents discharging of the input buffer B_1 (place P1) and filling of the output buffer B_2 (place P29). The input buffer contains 10 parts at time t=0 and one part from input buffer leaves each 2 seconds. The input buffer is empty at time t = 20 sec. At this time, the first part is added to output buffer and each two seconds the next part is added to the output buffer until it contains all 10 parts. Next figures illustrate behavior of the first part (or fourth part respectively) of the conveyor. Marking of P3 (P12) signifies that the first part (or fourth part respectively) is occupied. Since P3 and P4 form P-invariant, the marking of P3 is complement to the marking of P4 - if place P3 contains token, place P4 is empty. The first element becomes occupied from time t = 2 sec to t =22 sec, while the forth element becomes occupied from t = 8 sec to t = 28 sec.

In this example, we have shown also internal behavior of the conveyor, since it distinguishes two delays:

- the discretization of the continuous product quantity by the threshold of T1
- the transport delay on the conveyor (timing of T1, T3,…, T19).

This model is just an approximation of the continuous flow due to the discretization. The model is precise in discrete values of time ($0^+, 2^+, 4^+, \ldots 38^+$).

3.2 *Extended HPN model*

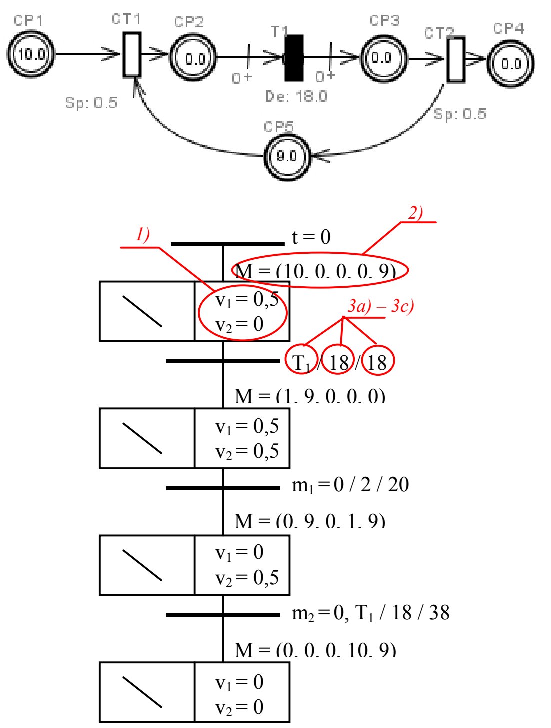

Fig. 4. - Extended HPN model of conveyor and its evolution graph

1) Instantaneous firing speed of T_j^c
2) Marking of P_i^c at the beginning of IB-state
3a) event, which caused passage from one IB-state to the other one
3b) relative time – the time elapsed in previous IB-state
3c) absolute time - the time elapsed from t=0

A conveyor model by extended HPNs is given in Fig. 4. Markings of continuous places CP1 and CP4 correspond to the number of parts in the input buffer B_1 and the output buffer B_2. Speeds of continuous transitions CT1 and CT2 correspond to the speed of the conveyor (the input speed is equal to the output speed). The maximal speeds associated to these transitions correspond to:

$$V1 = V2 = V * D\max = 0.5 * 1 = 0.5 \text{ kg/sec} \quad (4)$$

Place CP5 models the conveyor capacity. The delay associated to the transition T1 corresponds to time, which an infinitely small amount of product spends on the conveyor.

$$d1 = \frac{L}{V} = \frac{9}{0.5} = 18 \text{ sec} \quad (5)$$

The weight 0^+ represents infinitely small threshold of the transition T1. It means, that as soon as the infinitely small quantity of the product is put on the conveyor, the transition T1 becomes enabled and this quantity will reach the end of the conveyor, when time d_1 has elapsed.

3.3 *GBPN model*

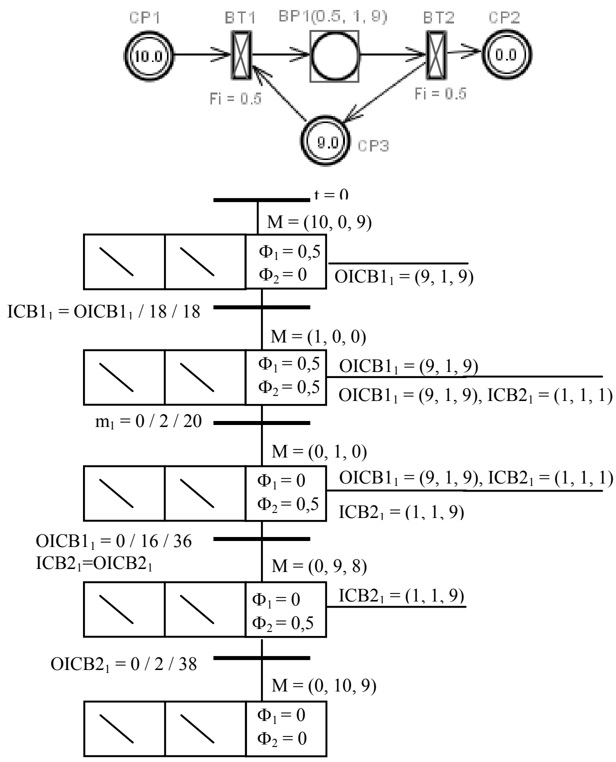

Fig. 5. – GBPN model of conveyor and its evolution graph

A conveyor model by GBPNs is shown in Fig. 5. Continuous places CP1 and CP2 correspond to input and output buffer. Batch place BP1 models the existence of the product on the conveyor. Batch function of BP1, which gives the speed of transfer, maximal density and the length of place, is {0.5, 1, 9}. The maximal speeds Φ_1 and Φ_2 associated to batch transitions BT1 and BT2 are given by equation (4).
The behavior is described by the evolution graph in Fig. 5. Each IB-state of GBPN consists of three

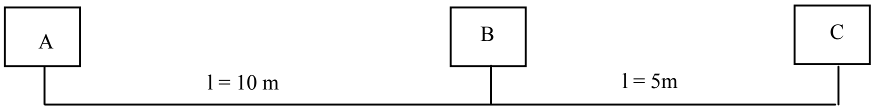

Fig. 6. - Three nodes A, B and C accessing a common bus

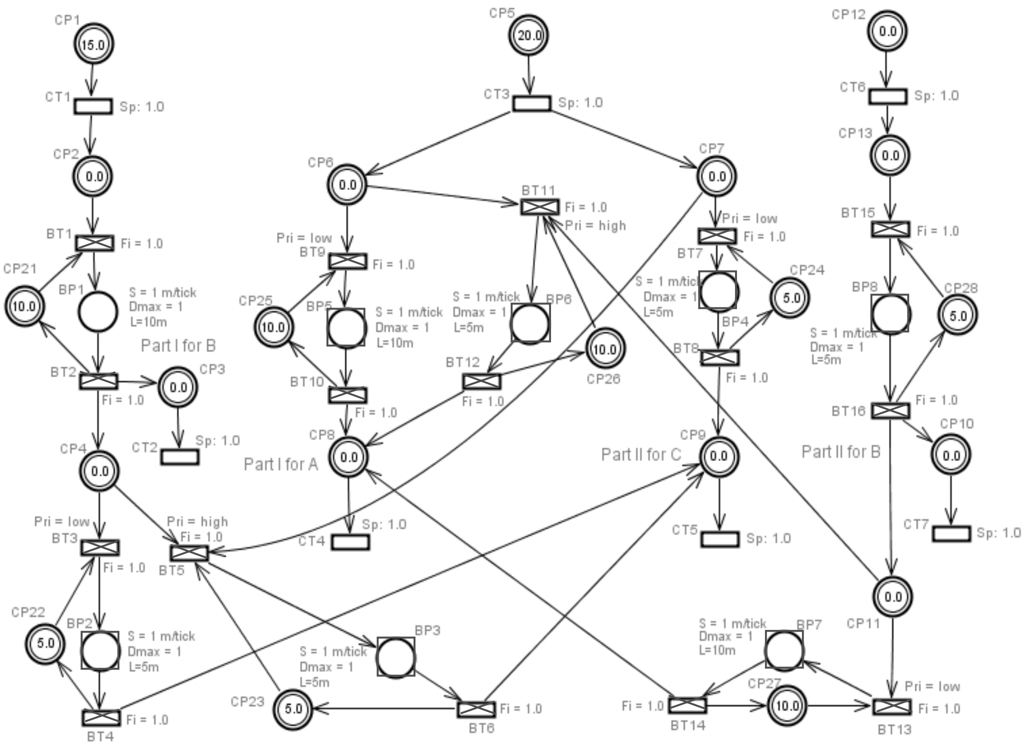

Fig. 7. – Physical layer model by GBPN

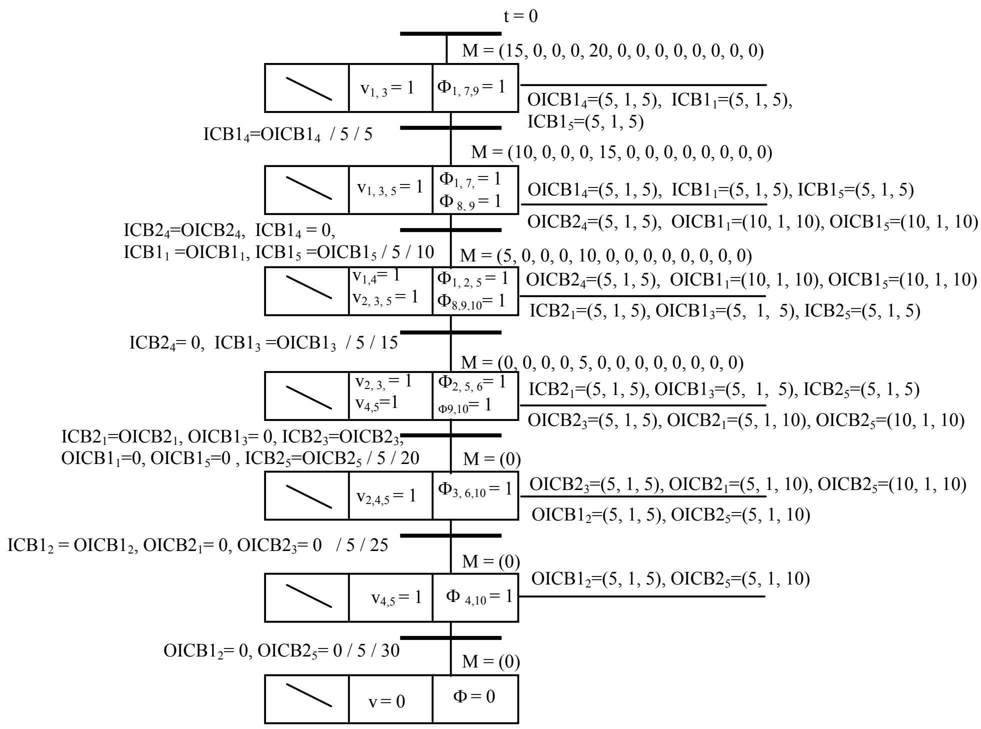

Fig. 8. – Evolution graph of model by GBPN

parts: discrete, continuous and batch part. The batch part contains constant speed of batch transitions during IB-state.

4. INDUSTRIAL EXAMPLE

4.1 Physical layer

Fig. 6 shows three nodes A, B and C accessing a common bus. The bus is one segment consisting of two parts – part I of length 10 meters between stations A and B, part II of length 5 meters between stations B and C. The aim is to model behavior of the physical media and to indicate the traffic on the bus to the Media Access Control sub-layer. In this example the nodes can transmit whenever they want at the speed 1 bit/tick. The signal propagates at the speed 1 meter per tick.

The physical media model by means of the batches Petri Nets is presented in Fig. 7. Continuous transition CT1 represents consecutive transmission of bit sequence from the node A. Batch transitions BT1, BT2, batch place BP1 and continuous place CP21 represent transmission delay on the part I. Traffic on the part I from the left side is indicated to the node B by CP3. Consecutively the bit sequence propagates on the part II with transmission delay 5 (BT3, BT4, BP2 and CP22). Continuous transition CT3 represents transmission of bit sequence from the node B to both directions, to the right and to the left. The transmission delay on part I is 10 ticks (BT9, BT10, BP5 and CP25) and 5 ticks on part II (BT7, BT8, BP4 and CP24). Similarly CT6 represents transmission from the node C with delay 5 ticks on the part II (BT15, BT16, BP8 and CP28) and 10 ticks on the part I (BT13, BT14, BP7 and CP27). When both the bit sequences from the node A and the bit sequences from the node B are propagated on the part II, then they collide and they form one bit sequence. The collided bit sequence is also transmitted with the delay 5 ticks (BT5 BT6, BP3 and CP23). In the same way the collided sequence from nodes B and C on the part I is transmitted with delay 10 (BT11, BT12, BP6 and CP26).
When the system is extended by a new node, the model grows only linearly - each part is represented by one transmission to the right side and one to the left side and one collided transmission to the right side and one collided transmission to the left side (with exception of the end parts).

Fig. 8 shows the evolution graph of GBPN in Fig. 7. Node A starts to transmit 15 bits and node B starts to transmit 20 bits at time 0. The node A detects the bus activity from the right side (corresponding to v_4) during $t \in <10,30)$. The node B detects the bus activity from the left side (corresponding to v_2) during $t \in <10,25)$ and from the right side (corresponding to v_7) there is no bus activity. The node C detects the bus activity from the left side (corresponding to v_5) during $t \in <5,30)$.

5. CONCLUSION

This article shows how the systems with the continuous flows delays can be modeled by extended hybrid Petri Nets. Two extensions have been used in this text – extended HPN by David and Caramihai (2000) and generalized Batches Petri Nets by Demongodin (2001). As illustrated on basic example, both extensions have similar expressive power with respect to modeling of continuous flows.

The physical layer example illustrates how this approach can be used to model the transmission delay on the communication media with bus topology. Indication of the activity on the bus is used by MAC sub-layer in order to decide when the transmission is allowed and when it is not allowed. Since the bit sequences are rather long represented by big amount of tokens, it is attractive to model the physical layer as continuous part of HPN and other layers as discrete part of HPN. The physical layer example in this article illustrates reasonable size of the HPN evolution graph.

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