Continuous Petri Nets and Polytopes*

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Abstract – *This article addresses the problem of the computation of instantaneous firing speed in Invariant Behavior state (IB-state) of Constant speed Continuous Petri Net (CCPN) with presence of actual conflicts. The adopted approach is based on polyhedral computations applied to specify an area of possible instantaneous firing speed. If the actual conflicts are resolved by global priorities, the instantaneous firing speed is found in a set of the polytop vertices or alternatively it is found by one formulation of the linear programming problem per each priority level. The approach shown in this article assumes the speed maximisation being prior to priority resolution.*

Keywords: Continuous Petri Nets, polytopes, linear programming, hybrid systems.

1 Introduction

The text is based on the Continuous Petri Net model presented by R. David and H. Alla [3]. These authors have obtained a continuous model by fluidization of a discrete Petri Net. Further Continuous Petri Nets constitute part of Hybrid Petri nets [4] made of a "continuous part" (continuous places and transitions) and a "discrete part" (discrete places and transitions). The continuous part can model systems with continuous flows and the discrete part models the logic functioning. Autonomous Continuous Petri Nets [10] and other models like DAE (Differential Algebraic Equations) Petri Nets [5], Batches Petri nets [6], First-Order Hybrid Petri Nets [7] have been studied intensively since this research area presents important bridge to hybrid systems (a bibliography on hybrid Petri Nets could be found at http://bode.diee.unica.it/~hpn). These models are subject of the algorithm development that might make use of polyhedral computations outlined in this text.

Instantaneous firing speed needs to be determined when *evolution graph* is constructed. This approach can be particularly useful when Continuous Petri Net is used to approximate discrete Petri Net, since evolution graph can represent the net behavior in very dense form. This article is motivated by the fact that the iterative algorithm finding instantaneous firing speed given in [3] may not be used when there is actual conflict.

The rest of this article is organized as follows: Section 2 surveys basic terms and it shows the algorithm determining enabled transitions. Section 3 presents the model where a speed maximisation is not assumed. It shows how the space of possible instantaneous firing speed in the free speed model can be determined by the polytop. Basic examples are given in order to convince the reader about utility of polyhedral computations in this area. Section 4 presents *maximum speed area* to which instantaneous firing speed has to belong, when the maximum speed model is assumed. Examples of actual conflicts show how this area is constituted form the polytop faces. Section 5 presents resolution of actual conflicts by priorities, it defines *priority determined speed* and it proposes two algorithmical solutions, one based on polytops and one based on linear programming.

2 Preliminaries

This section surveys some basic terms and algorithms from the area of CCPNs based on [3].

Definition 1 *A* constant speed continuous Petri Net *(CCPN) is a sextuple* $R = [P, T, V, Pre, Post, M(0)]$, *where*

- *The definitions of* $P, T, Pre, Post$ *are similar to those of discrete PNs. This article is only concerned with CCPNs with natural valued weights of arcs, however general case, where* real positive numbers *are associated with arcs, can also be considered.*
- $M(0)$ *is initial marking of continuous PN. It is a vector of positive or zero real numbers.* $M(t)$ *denotes the marking at time t.*
- $V : T \longrightarrow \mathcal{R}^+$ *is vector of maximal firing speeds;* V_j *denotes* maximal firing speed *of the transition* T_j.

The fact, that V_j is a constant (independent of marking and time), gives the name to this class of continuous PNs called constant speed continuous PNs. Further $v_j(t)$ denotes *instantaneous firing speed* of the transition T_j at a time t. Value of $v_j(t)$ is bounded by interval $\langle 0, V_j \rangle$ and since it is dependent on $M(t)$ it changes in separate IB-states of the evolution graph [3].

Definition 2 *A place P_i is* marked *at a time t if $M_i(t) > 0$.*

Definition 3 *A transition T_j is* strongly enabled *at a time t if all places P_i of ${}^oT_j$ are marked.*

Source transition is supposed to be strongly enabled, in accordance with definition 3.

Definition 4 *A place P_i is* supplied *at a time t if there is at least one transition T_j in ${}^oP_i$, which is enabled (strongly or weakly).*

Definition 5 *A transition T_j is* weakly enabled *at a time t if there is a place P_i of ${}^oT_j$, which is not marked and it is supplied, and remaining places of ${}^oT_j$ are either marked or supplied.*

And finally we suppose the transition to be enabled at a time t if it is strongly enabled or weakly enabled.

The recursive definitions of supplied place and weakly enabled transition does not allow direct determination of supplied places and weakly enabled transitions.

Calculation of the set of enabled transitions for a given marking is given by Algorithm 1 in [3]. This algorithm is based on iterative upgrade of vectors A (one bit assigned to each place) and E (one bit assigned to each transition). This iterative algorithm converges in polynomial time.

Definition 6 *The* balance *of P_i in CCPN is:*

$$B_i(t) = \sum_{T_j \in {}^oP_i} Post(P_i, T_j) \cdot v_j(t) - \sum_{T_k \in P_i^o} Pre(P_i, T_k) \cdot v_k(t) \quad (1)$$

The balance of P_i at a time t corresponds to the derivative of its marking, i.e., $m_i'(t) = B_i(t)$ and

$$m_i(t + dt) = m_i(t) + B_i(t) \cdot dt \quad (2)$$

If the balance of P_i is positive the marking m_i increases and if the balance is negative the marking m_i decreases.

Algorithm calculating instantaneous firing speed of enabled transition in [3] is based on iterative approach (i.e. $v_j^{r+1}(t)$, the value of instantaneous firing speed at iteration step $r+1$, is dependent on $v_j^r(t)$, the value from previous step). On the contrary, the approach presented in this article is based on the analytical determination of subspace of instantaneous firing speeds.

In order to simplify specification of instantaneous firing speeds constraints we give:

Definition 7 *A place P_i is* supplying *if there is at least one transition T_j in P_i^o , which is enabled (strongly or weakly).*

3 Free speed CCPN

CCPN are assumed to function at *maximum speed* [3], therefore $v_j(t)$ of strongly enabled transition T_j is equal to V_j and $v_j(t)$ of weakly enabled transition is the maximum possible one. This classical model will be called *maximum speed CCPN* in this article.

On the other hand, the systems not functioning at their maximum speed are also very interesting subject of research (e.g. for verification problems). Therefore this section is devoted to the analysis of *free speed CCPN* (the model is formally given by the set of speed constraints related to the system of inequalities (3), (4), (5)). The term "free speed" is linked to the fact that $v_j(t)$ of strongly enabled transition T_j is not explicitly equal to V_j and $v_j(t)$ of weakly enabled transition is not explicitly the maximum possible one. This model can be used when one describes the systems with undetermined but bounded firing speed (in the limit case, when $V_j = \infty$ for all $T_j \in T$, the free speed CCPN is identical to the autonomous CPN). In this article the "free speed CCPN" model is used as an intermediate step illustrating the subspace of instantaneous firing speeds. The subspace is further used to determine the instantaneous firing speeds in "maximum speed CCPN" given in section 4.

The instantaneous firing speed $v_j(t)$ of enabled transition T_j at a time t is bounded on interval $\langle 0, V_j \rangle$ as the firing speed cannot be negative and it cannot be higher than the maximal firing speed (see inequalities (3) and (4)).

Further $v_j(t)$ is dependent on the marking of $P_i \in^o T_j$. There is no restriction issued by marked place P_i, since $v_j(t)$ is supposed to be finite and $|P_i^o|$ is finite as well. As a consequence the marked place P_i remains marked at least for a short time interval, even if its balance B_i is negative. The situation is different for unmarked place. As the marking of the place P_i cannot be negative, $m_i(t+dt) \geq 0$, the balance $B_i(t)$ of unmarked place has to be positive or zero due to equation (2). Among unmarked places only the set of supplying places will be considered (e.g. P_2 in Figure 1) since the non supplying ones have no influence on any instantaneous firing speed. (Please notice: if P_i is unmarked supplying, then there exists weakly enabled T_j in P_i^o and therefore P_i has to be supplied).

As mentioned above, the subspace of instantaneous firing speeds for *free speed CCPN* , is constrained by

the following system of inequalities (3), (4), (5):
Speed limits of enabled transitions. According to the definition 1, an instantaneous firing speed must not be grater than its specified maximal value:

$$v_j(t) \le V_j \quad \forall j \text{ such that } T_j \text{ is enabled} \tag{3}$$

Non-negative speeds of enabled transitions. An instantaneous firing speed must be positive or zero:

$$v_j(t) \ge 0 \quad \forall j \text{ such that } T_j \text{ is enabled} \tag{4}$$

Non-negative balances of unmarked supplying places. If there is unmarked supplying place P_i then there exists at least one weakly enabled transition in P_i^o, and then the balance of P_i must be positive or zero:

$$B_i(t) \ge 0 \quad \forall i \ P_i \text{ is supplying, and } m_i(t) = 0 \tag{5}$$

All variables dependent on time, $v(t), B(t), M(t)$, will be denoted simply v, B, M in the rest of this article, since they are used when a specific time is assumed.

Each B_i can be written as linear combination of instantaneous firing speeds of enabled transitions, due to equation (2). As a consequence the system of inequalities (3), (4), (5) can be written in the form, where instantaneous firing speeds of enabled transitions (denoted x in Polyhedral Computations) are the only variables.

Let c denotes the number of unmarked supplying places, d denotes the number of enabled transitions and k denotes index of enabled transition ranging from 1 to d. The subspace Π of $\mathcal{R}^d$ is *convex polyhedron* [8, 1] since Π is the set of solutions to the above mentioned finite system of inequalities (exactly there are $2d + c$ inequalities). Π is *convex polytop*, as it is convex polyhedron and it is bounded (due to (3) and (4)). One given point $x \in \Pi$ corresponds to the instantaneous firing speed vector v. If T_j is enabled then v_j is equal to x_k, the k-th coordinate of this point. Otherwise v_j is equal to zero.

Let Π be a convex polytop of $\mathcal{R}^d$. For a real d-vector a and a real number b, a linear inequality $a^T x \le b$ is called *valid* for Π if $a^T x \le b$ holds for all $x \in \Pi$. A subset F of a polyhedron Π is called a *face* of Π if it is represented as:

$$F = \Pi \cap \{x; a^T x = b\} \quad \forall \text{ valid inequality } a^T x \le b \tag{6}$$

The faces of dimension 0,1,d-1 are called *vertices, edges* and *facets*, respectively. The vertices coincide with *extreme points* of Π. The extreme point is defined as point which cannot be represented as convex combinations of two other points in Π.

Example 1:

The system of inequalities (3), (4), (5) for a simple CCPN in Figure 1 is shown below as system of 5 inequalities, since there are two enabled transitions T_1, T_2 and one unmarked supplying place P_2. Corresponding polytop Π is given only by 4 inequalities since the third inequality is redundant (the corresponding edge degenerated to vertex, as the corresponding inequality is just on the border of redundancy).

$$\begin{aligned} v_1 &\le 2 && \textit{edge } T_1 \textit{ in Figure } 1 \\ v_2 &\le 1 && \textit{edge } T_2 \textit{ in Figure } 1 \\ v_1 &\ge 0 && \textit{redundant } - \textit{vertex } T_{01} \textit{ in Figure } 1 \\ v_2 &\ge 0 && \textit{edge } T_{02} \textit{ in Figure } 1 \\ v_1 - v_2 &\ge 0 && \textit{edge } P_2 \textit{ in Figure } 1 \end{aligned} \tag{7}$$

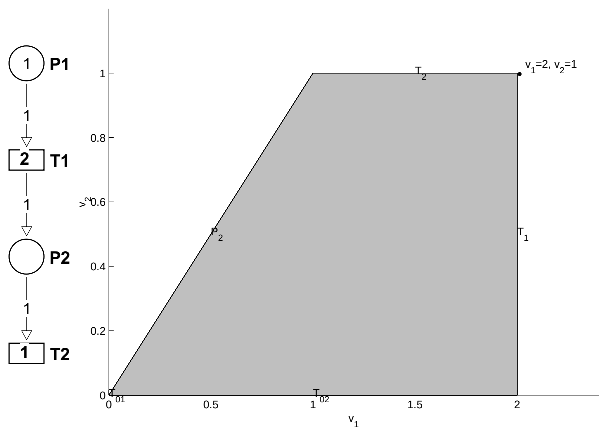

Figure 1: CCPN consisting of two transitions with different maximal firing speeds and polytop Π representing possible instantaneous firing speeds of the free speed model

Example 2:

Figure 2 illustrates parallelism. Polytop Π in Figure 3 determines possible instantaneous firing speeds. Particular triple of instantaneous firing speeds v_1, v_2, v_3 is given as particular point in Π.

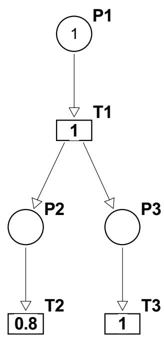

Figure 2: CCPN containing parallel branches with different maximal firing speeds

Upper bound of v_1 is given by a hyperplane corresponding to inequality $v_1 \le V_1$ and in 3-dimensional space it is *geometrically represented by a plane. This face of Π corresponding to upper bound of v_1 is the facet (of dimension d-1=2) labeled as T_1 in Figure 3. Similar applies for V_2, upper bound of v_2, and corresponding facet T_2. The situation is different in the case of T_3 as V_3 in fact does not limit v_3. In accordance to that a hyperplane corresponding to inequality $v_3 \leq 1$ is redundant. Since this hyperplane is just on the border of redundancy (it would not be redundant if V_3 will be decreased a little bit) it is possible to find a corresponding face of Π. There would be no face T_3 if V_3 will be increased, since corresponding hyperplane would not have any intersection with Π in such case. This face (labeled T_3) is not facet but it is "only" edge due to the fact that it is on the intersection of the facets T_1 and P_3.*

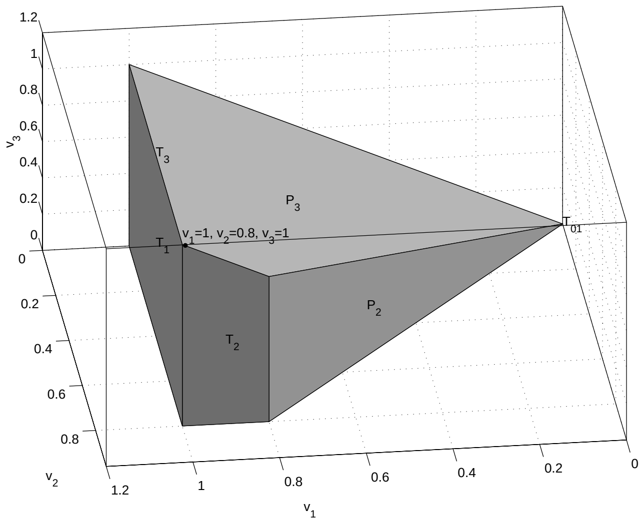

Figure 3: Polytop Π representing possible instantaneous firing speeds of free speed CCPN in Figure 2

The facet P_3 is determined by a hyperplane given by $Post(P_3, T_j)$ and $Pre(P_3, T_j)$. And the same applies for a facet P_2 which corresponds to the second unmarked place P_2. Lower bounds of v_2 and v_3 corresponds to the facets labeled T_{02} and T_{03} respectively (these two facets of Π are not visible in Figure 3 due to the orientation and non transparency of Π). Non-negative value of v_2 and v_3 implies non negative value of v_1 (i.e. inequality $v_1 \geq 0$ is redundant). Consequently face T_{01} is "only" vertex since it is intersection of facets T_{02}, T_{03}, P_2 and P_3.

During the construction of evolution graph for given CCPN we are able to derive the system of inequalities (3), (4), (5). This is in fact a (halfspace) *H-representation* of the polytop Π which can be given also by the *set of vertices, so called V-representation.* The transformation of *H-representation* to *V-representation* is known as *vertex enumeration* and opposite transformation as *facet enumeration* (the *facet enumeration* reduces to the *convex hull problem*, as Π is bounded). For more details like upper bounds on the numbers of faces and complexity of enumeration algorithms please refer to [8, 1].

Specific values of maximal firing speeds in this article are usually chosen in such way that there are few redundant inequalities in order to illustrate various behavior of CCPN in 3-dimensional space.

Example 3:

Figure 4 illustrates conflict. Corresponding polytop shows, that any possible instantaneous firing speed v_1 can grow up to its upper bound V_1 as in previous example. On the other hand, v_3 can reach its upper limit V_3 (facet T_3) only in the area where $v_1 - v_2 \geq V_3$ holds. In the remaining area ($v_1 - v_2 < V_3$) any possible instantaneous firing speed v_3 can grow only up to $v_1 - v_2$ (facet P_2). In the other words, growth of v_3 is not limited only by preceding transitions but also by v_2, which does not precede T_3. This is important difference to previous examples and it is caused by shared resource (conflict place P_2 in Figure 4).

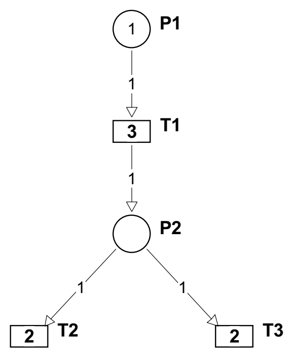

Figure 4: CCPN with conflict

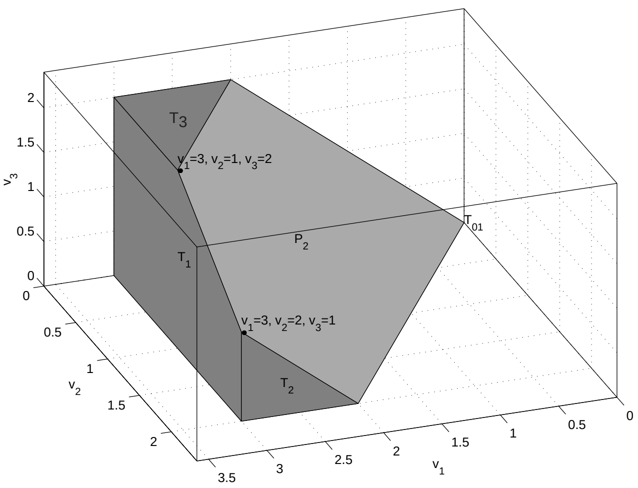

Figure 5: Polytop Π representing possible instantaneous firing speeds corresponding to free speed CCPN in Figure 4

In the terms of polytop in Figure 5, due to the conflict between transitions T_2 and T_3 there exists such plane given by constant speed of remaining transition T_1 (e.g. $v_1 = 2.5$) that by intersection of this plane with facet P_2 we obtain a line segment with negative slope (upper limit for v_3 decreases when v_2 increases, since $v_2+v_3 = 2.5$ on this line segment).

In the terms of the system of inequalities (3), (4), (5), if there is a structural conflict between two transitions T_a and T_b we can find such 2-dimensional subspace in Π given by constant speeds of remaining transitions that there is at least one non-redundant inequality in (5) of the form $\alpha_a v_a + \alpha_b v_b + \beta \geq 0$ and $\alpha_a < 0, \alpha_b < 0$. This inequality corresponds to the balance of the conflict place P_i. The constants α_a, α_b are related to the net structure ($\alpha_a = Post(P_i, T_a) - Pre(P_i, T_a)$, $\alpha_b = Post(P_i, T_b) - Pre(P_i, T_b)$) and their negativeness is founded on fact that each of the transitions T_a and T_b takes more than it gives to the conflict place P_i.

Approach given in this section is very illustrative when studying possible basic behaviors of free speed CPPNs, but it is not sufficient when CCPN is used to model existing system with deterministic behavior.

4 Maximum firing speed

As many existing systems run at their maximal speeds, it is attractive to define their characteristics.

Maximum speed of any strongly enabled transition T_j can be determined directly as $v_j = V_j$. Consequently new *polytop* Θ of $\mathcal{R}^w$ (w = *number of weakly enabled transitions*) can be obtained by *reduction* of the polytop Π of $\mathcal{R}^d$ ($d = w+$*number of strongly enabled transitions*). This reduction is done by changing each inequality, corresponding to strongly enabled transition in the subsystem (3), to equality (as a consequence, the inequality is redundant). Since each equality can be written as two inequalities (e.g. $v_j = V_j$ can be written as $v_j \leq V_j$ and $v_j \geq V_j$), the polytop Θ is convex and bounded as required by definition.

The maximum speed area G is subset of Θ, since Θ is specific subset of Π such that above mentioned reduction does not eliminate any part of G.

Let us assume the CCPN in Figure 2 to run at its maximum speed. Corresponding polytop Θ can be determined as intersection of Π in Figure 3 with plane $v_1 = 1$, corresponding to strongly enabled transition T_1. As there is just one strongly enabled transition, the intersection corresponds to the facet T_1. Please notice that speed limit inequalities (3) of strongly enabled transitions are never redundant since their input places are marked.

And finally, we can determine polytop Θ as facet T_1 also in Figure 5, since it is obtained as intersection of the polytop Π with the plane $v_1 = 3$, corresponding to strongly enabled transition T_1. The polytop Θ is shown in Figure 6.

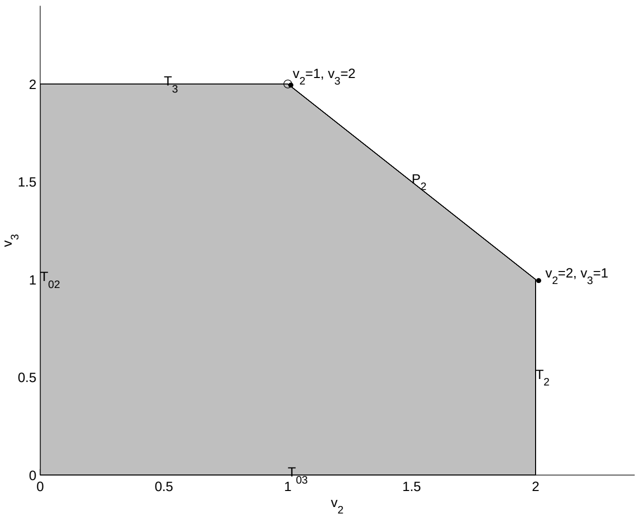

Figure 6: Polytop Θ used to determine instantaneous firing speeds of weakly enabled transitions in Figure 4

Both polytopes Π and Θ can be used, when studying maximum speed CCPN. Since $dim(\Theta) \leq dim(\Pi)$, only polytop Θ will be used in the following examples in order to demonstrate various cases in 3-dimensional space.

Definition 8 *A possible instantaneous firing speed* $v = [v_1, \ldots, v_k \ldots, v_d]$ *(*$v \in \Pi$ *in the sense of the polytop given by the system of inequalities (3), (4), (5)) is said to be* possible maximum speed *if there does not exist any* $u \in \Pi$ *such that* $u_k \geq v_k$ *for all* $k = 1 \ldots d$.

Definition 9 *A subset* G *of a polyhedron* Π *is called a* maximum speed area *of* Π *if each* $v \in G$ *is possible maximum speed.*

Property 1: If there is no structural conflict (see Figures 2) then there is no nondeterminism in maximum speed CCPN and maximum speed area is always one vertex of corresponding polytop Θ.

This unique vertex of polytop Θ can be found in polynomial time by one call of linear programming [9] aiming at maximisation of objective function $J = s^T x$, with arbitrary nonzero positive finite entries of s, i.e. $s_j \in (0, \infty)$ for all $j = 1 \ldots w$.

Due to the definition 8 it is obvious that the maximum speed area G is a subset of the set of all faces of polytop Θ (the set is called *face poset*), since no interior point of a convex polytop can reach maximum value of any convex objective function. In order to determine the maximum speed area it is sufficient to check all faces of polytop Θ.

Property 2: Let F_q be a face of Θ of dimension $k \geq 1$. All points $x \in F_q$ belong to the maximum speed area if and only if all faces $F_{k-1} \subset F_q$ belong to the maximum speed area.

When recursively applying Property 2 one can derive:

Property 3: Face F_q of Θ belongs to maximum speed area (i.e. all points $x \in F_q$ belong to the maximum speed area) if and only if all vertices $F_0 \subset F_q$ belong to the maximum speed area.

Due to Property 3, the maximum speed area G is fully determined by the set of vertices belonging to G. One can simply determine whether a vertex belongs to the maximum speed area G by the *vertex enumeration* and by the selection of vertices satisfying definition 8.

This procedure is applied to all examples in this article. A vertex belonging to the maximum speed area is indicated by small dot, labeled by instantaneous firing speed. For example Figure 5 has two vertices of this kind $[v_1 = 3, v_2 = 1, v_3 = 2]$ and $[v_1 = 3, v_2 = 2, v_3 = 1]$, since the speed maximisation does not specify deterministic behavior of maximum speed CCPN given in Figure 4. If the value of V_1 would be raised up to 4, then the facet T_1 will be moved to the left and maximum speed area will consist of only one vertex $[v_1 = 4, v_2 = 2, v_3 = 2]$ and behavior of maximum speed CCPN will be deterministic. This is due to the fact, that Property 1 is implication but not equivalence (existence of structural conflict is necessary but not sufficient condition for existence of actual conflict).

Definition 10 *Let $K = [P_i, \{T_j, T_k\}]$ be a structural conflict. There is an* actual conflict *between transitions T_j and T_k if there are at least two possible maximum speeds v and v' such that $v_j < v'_j$ and $v_k > v'_k$.*

Informally we say that there is actual conflict between two transitions when possible increase of instantaneous firing speed of one transition must be compensated by the decrease of instantaneous firing speed of the other transition.

Property 4: If there exists a face $F_q \in G$ such that $k \geq 1$ then there exists an actual conflict.

This property can be proved as follows:
F_q is not vertex since $k \geq 1$ (i.e. F_q is at least edge)
$\Rightarrow$ there exist at least two vertices $v \in F_q$ and $v' \in F_q$ belonging to G
$\Rightarrow$ there exist two transitions T_j and T_k such that $v_j < v'_j$ and $v_k > v'_k$ (otherwise v, w could not satisfy definition 8)
$\Rightarrow$ actual conflict exists

Property 5: If there exists an actual conflict then there exists a face $F_q \in G$ such that $k \geq 1$.

This property can be proved as follows:
due to the actual conflict, there exist two transitions T_j and T_k such that there are two possible maximum speeds (not necessarily vertices) v and v' such that $v_j < v'_j$ and $v_k > v'_k$
$\Rightarrow G$ consists of at least two points v and v', since they satisfy definition 8 and the polytop is convex by definition
$\Rightarrow$ either both v and v' belong to one face $F_q \in G$ such that $k \geq 1$ or each of them belongs to a distinct face $F_q \in G$ such that $k \geq 1$

Property 6: There is actual conflict between transitions T_j and T_k when the following three conditions are satisfied:

- (a) there is structural conflict $K = [P_i, \{T_j, T_k\}]$ and
- (b) M_i =0 and consequently T_j, T_k are weakly enabled transitions (v_j, v_k are coordinates of Θ)
- (c) a hyperplane, given by inequality (5) corresponding to P_i, intersects with at least two points belonging to the maximum speed area G.

Example 4:

Figure 7 illustrates structural conflicts $K_1 = [P_2, \{T_2, T_3, T_4\}]$ and $K_2 = [P_3, \{T_3, T_4\}]$. Maximum speed area G in Figure 8 is one face of dimension 2 (facet P_2 given by vertices $[v_2 = 1, v_3 = 0.33, v_4 = 0.33]$, $[v_2 = 0.3, v_3 = 0.8, v_4 = 0.1]$, $[v_2 = 0.4, v_3 = 0.8, v_4 = 0]$, and $[v_2 = 1, v_3 = 0.5, v_4 = 0]$) and one face of dimension 1 (edge given by vertices $[v_2 = 1, v_3 = 0.33, v_4 = 0.33]$, $[v_2 = 1, v_3 = 0.2, v_4 = 0.4]$. Both structural conflicts lead to actual conflict. Using this example we can illustrate Property 3: the facet P_3 does not belong entirely to G since vertices $[v_2 = 0, v_3 = 0.8, v_4 = 0.1]$, $[v_2 = 0, v_3 = 0.2, v_4 = 0.4]$ does not belong to G. Please notice that the maximum speed area G is not convex.

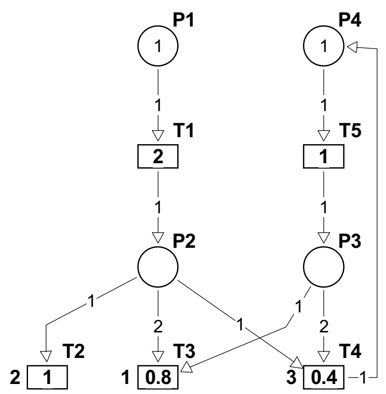

Figure 7: CCPN with two actual conflicts

5 Resolution of actual conflicts by priorities

Deterministic behavior of maximum speed CCPN is not given when actual conflict is present. Therefore we propose a global priority assignment defined in the following way:

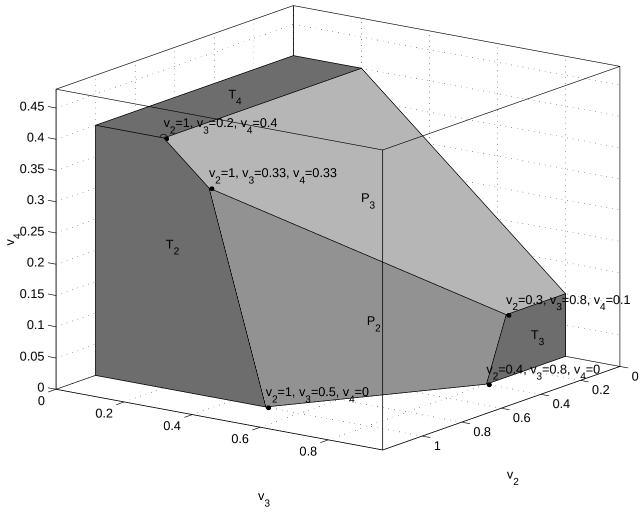

Figure 8: Polytop Θ used to determine instantaneous firing speeds of weakly enabled transitions in Figure 7

Definition 11 *Let* $R = [P, T, V, Pre, Post, M(0)]$ *be a maximum speed CCPN. A* maximum speed CCPN with global priorities *is a seven tuple* $R' = [P, T, V, Q, Pre, Post, M(0)]$ *where:*

- *The definitions of* $P, T, V, Pre, Post, M(0)$ *are similar to those in CCPN.*
- $Q : T \longrightarrow \{0, \mathcal{N}^+\}$ *is vector of global priorities;* Q_j *denotes* global priority *of the transition* T_j *in the sense that* T_j *has higher priority than* T_l *if and only if* $Q_j > Q_l$.

Definition 11 allows two transitions T_j and T_l to have the same priority $Q_j = Q_l$, so one can use the term *priority level*, to which transitions with equal priority are associated. Label on the left side of the transition designates its priority (see Figure 7). Priority is 0 (lowest priority), if there is no number on the left side of the transition.

Deterministic behavior of maximum speed CCPN with global priorities R' is given by the choice of one possible maximum speed in the maximum speed area G.

Definition 12 *A possible maximum speed* $v = [v_1, \ldots, v_j \ldots, v_d]$, $v \in G$, *is said to be* priority determined speed *if for any* $u \in G$ *and for any* T_j *such that* $v_j < u_j$ *there exists some* T_k *such that* $Q_k \geq Q_j$ *and* $v_k > u_k$.

Definition 13 *A subset* H *of* G *is called a* priority determined area *of* Π *if each* $v \in H$ *is priority determined speed.*

There are several priority determined speeds if actual conflicts are not resolved. Assume priority assignment $Q = [0, 2, 1, 1, 0]$ in Figure 7 leading to priority determined area given by edge ($[v_2 = 1, v_3 = 0.2, v_4 = 0.4]$, $[v_2 = 1, v_3 = 0.33, v_4 = 0.33]$) and by edge ($[v_2 = 1, v_3 = 0.5, v_4 = 0]$, $[v_2 = 1, v_3 = 0.33, v_4 = 0.33]$).

Priority determined area is just one priority determined speed (one vertex of Θ) if all actual conflicts are resolved by priorities (for example see Figure 7 where priority assignment $Q = [0, 2, 1, 3, 0]$ leads to priority determined speed $[v_2 = 1, v_3 = 0.2, v_4 = 0.4]$). In such case *there are two algorithmic solutions*:

(I) **Vertex enumeration**. To enumerate vertices of Θ, then to select vertices determining maximum speed area G using definition 9, then to choose the priority determined speed using definition 12. There does not exist any polynomial bound for this algorithm since the number of vertices of Θ is proportional to 2^w (w is the number of weakly enabled transitions).

(II) **Linear programming**. To find the maximum speed by calling linear programming for each transition. Iterations are executed in the order given by the transitions priorities. First, one partial solution S is found (for highest priority T_j) by linear programming aiming at maximisation of objective function $J = v_j$ subject to Θ. Then new equation $v_j = S_j$ is added to the system of inequalities (corresponding new polytop Θ' is intersection of Θ with equation $v_j = S_j$). Then algorithm repeats for next transition having equal or lower priority. Further all transitions are proceeded in a similar way and the final solution S determines the instantaneous firing speed satisfying priority order.

6 Conclusion

This article addresses the problem of the computation of instantaneous firing speed for given IB-state of CCPN. Two algorithmic solutions have been shown. The *vertex enumeration* solution is based on the analytical determination of the subspace of instantaneous firing speeds. The nondeterministic "free speed CCPN" model is used as an intermediate step to illustrate influence of speed and balance constraints on the subspace. The subspace is further used to determine the maximum speed area G in "maximum speed CCPN" (classical CCPN model used by other authors). If there is no actual conflict the instantaneous firing speed is determined directly, since the maximum speed area G is just one vertex (see Properties 4 and 5). Otherwise specific vertex in G is chosen up to the transition priorities. The vertex enumeration solution is very illustrative (namely if there are less than 3 weakly enabled transitions), but there is no polynomial bound on the algorithm execution time.

While using *linear programming* solution the instantaneous firing speed can be found by one formulation of the linear programming problem if there is no actual conflict in given CCPN. Otherwise the actual conflicts have to be resolved by global priorities, and the instantaneous firing speed is found by one formulation of the linear programming problem per each priority level. Prior to using this solution, it is "safer" to resolve all structural conflicts by priorities since the linear programming solution is not able to detect existence of actual conflicts for given CCPN. The linear programming solution is polynomial.

Even if the vertex enumeration solution seems to be less efficient it is very interesting since it distinguishes influence the three phenomena (constrains, maximisation, priorities). Consequently the subspace representation by polytopes can be used to compare the approach adopted in this article with other approaches (see the next paragraph). Moreover the state space representation using polyhedral computations can be applied to solve other problems. For example an existence or non-existence of polytop can be studied in verification problems where positive lower bound on instantaneous firing speed is considered (in such case the set of inequalities (4) changes to $v_j(t) \geq X_j$ where $X_j < V_j$). Another example is representation of the autonomous CCPN state space by convex cone (in such case V_j is set to ∞ for all $T_j \in T$ in the set of inequalities (3), so this set can be eliminatd; consequently the polyhedron Π is not bounded and it is given by rays).

The approach shown in this article assumes the speed maximisation being prior to priority resolution, since $H \subset G$ by definition of H.

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